Stellar/Planetary Atmospheres

Part 11: continuous opacities

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Topics

- continuous opacities
    - H I
    - H^-
    - metals
    - electron scattering
    - molecular opacities

continuous opacities

- must know all opacities in order to solve RTE and compute spectrum
- these are functions of T, $P_{\rm gas}$, P_e, λ, and abundances
- caution: strictly speaking also of the radiation field!

continuous opacities

- distinguish between
  - 'slowly' varying continuous opacities (but discontinuities)
  - 'rapidly' varying line opacities (dex over 1 Å!)
- 2 categories for continuum opacities:
  1. b-f transitions: photoionizations
  2. f-f transitions: charged particle (electron) accelerated in E-field of other charged particle (ion) and a photon is absorbed
- continuous scattering processes:
  1. Thompson (electron) scattering
  2. Rayleigh scattering

stimulated emission

- typically not included in f-f and b-f transition data!
- in LTE, it's simply given by

$$(1 - \exp(-h\nu/kT))$$

- depending on λT, this can be a big correction:
- $\lambda T = 0.2 \rightarrow \Delta = 0.08\%$ ($\lambda = 2000\,\text{Å}$ for $T = 10^4\,\text{K}$)
- $\lambda T = 0.4 \rightarrow \Delta = 2.8\%$ ($\lambda = 4000\,\text{Å}$ for $T = 10^4\,\text{K}$)
- $\lambda T = 0.6 \rightarrow \Delta = 10\%$ ($\lambda = 6000\,\text{Å}$ for $T = 10^4\,\text{K}$)
- $\lambda T = 0.8 \rightarrow \Delta = 19.8\%$ ($\lambda = 8000\,\text{Å}$ for $T = 10^4\,\text{K}$)
- $\lambda T = 1.0 \rightarrow \Delta = 31.1\%$ ($\lambda = 10000\,\text{Å}$ for $T = 10^4\,\text{K}$)

general stuff

- opacities written as

$$\kappa(\lambda) = \sum \sigma_i(\lambda) n_i$$

- n_i from EOS
- $\sigma_i(\lambda)$: cross section for, e.g., photoionization out of level i
- $\sigma_i(\lambda)$ from QM or measurements

H I b-f

- also valid for H I like ions!
- basic picture:

Table 8.2. *Absorption edges.*

n	λ, Å	Name
1	912	Lyman
2	3647	Balmer
3	8206	Paschen
4	14588	Brackett
5	22790	Pfund

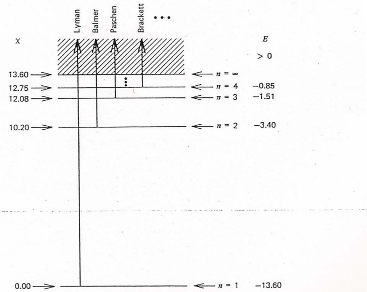


Fig. 8.1. This simplified energy level diagram for hydrogen shows the quantum numbers, n, binding energy, E, and the excitation potential, χ, in electron volts for the first four levels and the continuum.

H I b-f

- b-f cross section can be computed with QM:

$$\sigma_n(\lambda) = \frac{32}{\sqrt{27}} \frac{\pi^2 e^6}{h^3 c^3} R \frac{\lambda^3}{n^5} g_n'(\lambda) \equiv \alpha_0 \frac{\lambda^3}{n^5} g_n'(\lambda)$$

- $R = 1.0968 \times 10^5$cm: Rydberg constant for H I
- $\alpha_0 = 1.044 \times 10^5$ (λ in Å)

H I b-f

- g_n' is the *Gaunt factor*
- introduced to bring QM result into same form as classical (Kramer's) formula
- Gaunt factors have been calculated by, e.g., Karzas+Latter, ApJSupp, 6:167 (1961)
- often used: fit formulae, e.g.,

$$g_n'(\lambda) = 1 - 0.3456(\lambda R)^{-1/3}\left(\frac{\lambda R}{n^2} - 0.5\right)$$

H I b-f cross sections

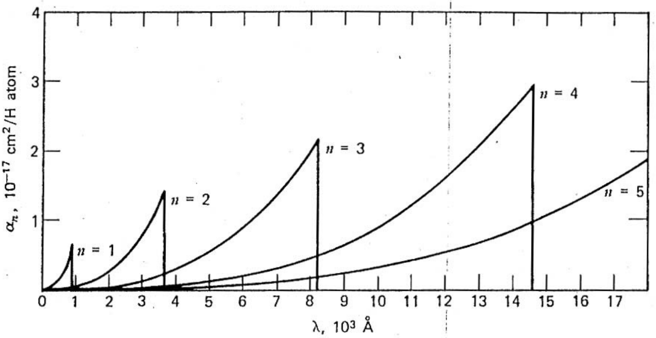


Fig. 8.2. The bound–free absorption coefficient for hydrogen increases with n.

H I b-f

- $\approx$ small contributions of levels $n > n_T = 3 \ldots 6$ by integration:

$$\sum_{n_T}^{n_{\max}} \frac{1}{n^3} \exp(-\chi_n/kT) \approx -\frac{1}{2} \int_{n_T}^{\infty} \exp(-\chi/kT)\, d(1/n^2)$$

- with $d\chi = -I_H\, d(1/n^2)$

$$= \frac{1}{2} \int_{\chi_T}^{I} \exp(-\chi/kT)\, \frac{d\chi}{I_H}$$

$$= \frac{kT}{2I_H} \left[\exp\left(-\frac{\chi_T}{kT}\right) - \exp\left(-\frac{I_H}{kT}\right) \right]$$

H I b-f

- with $\Theta = 5040/T$ this can be written as

$$\kappa_{\mathrm{HIbf}} = \alpha_0 N_H \left[\sum_1^{n_T} \frac{\lambda^3}{n^5} g_n'(\lambda) 10^{-\chi\Theta} + \frac{\log_{10}(e)}{2\Theta I_H} \left(10^{-\chi_T\Theta} - 10^{-I_H\Theta} \right) \right]$$

H I f-f

- f-f absorption is much smaller
- but important at large λ and/or large T
- collision electron-proton $\rightarrow$
- photon can be absorbed, increasing electron energy
- probability depends on relative speed
- classical differential cross section (cm^2 per hydrogen atom):

$$\frac{d\sigma_{\mathrm{ff}}}{dv} = \frac{2}{\sqrt{27}} \frac{h^2 e^2 R}{\pi m^3} \frac{1}{\nu^3 v}$$

H I f-f

- total cross section by integrating over all electron v's
- Maxwell distribution $\rightarrow$

$$\sigma_{\mathrm{ff}}(\lambda) = \frac{2}{\sqrt{27}} \frac{h^2 e^2 R}{\pi m^3} \frac{1}{\nu^3} \int_0^{\infty} \sqrt{\frac{2}{\pi}} \left(\frac{m}{kT}\right)^{3/2} \exp\left(-\frac{1/2 m v^2}{kT}\right) dv$$

- so that

$$\sigma_{\mathrm{ff}}(\lambda) = \frac{2}{\sqrt{27}} \frac{h^2 e^2 R}{\pi m^3} \frac{1}{\nu^3} \sqrt{\frac{2m}{\pi kT}}$$

H I f-f

- QM calculation (Gaunt, 1930) gives basically the same result
- but adds a f-f Gaunt factor:

$$\sigma_{\mathrm{ff}}(\lambda) \rightarrow \sigma_{\mathrm{ff}}(\lambda) g_{\mathrm{ff}}(\lambda)$$

- with (λ in Å)

$$g_{\mathrm{ff}}(\lambda) = 1 + 0.3456(\lambda R)^{-1/3} \left(\frac{\lambda k T}{h c} + 0.5 \right)$$

$H-$

- $H-$ has one *bound* state
- ionization potential $0.754\,\mathrm{eV}$ (16444 Å)
- not any more bounds states!
- $\rightarrow$ no spectral lines
- but important opacity source in the Sun!

H^- b-f

- polynomial fits to reproduce λ dependence of H^- b-f
- accuracy better than 0.2%
- H^- has autoionizing (unstable) levels above its ionization limit
- $\rightarrow$ resonances in the absorption
- $+$ forbidden H^- continuum
- these are very small

H^- b-f cross sections

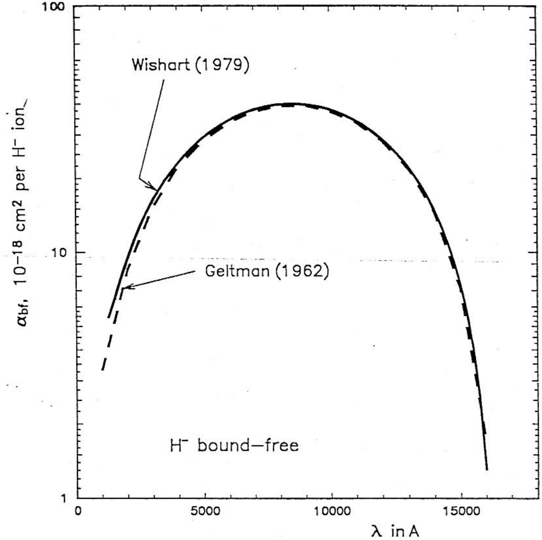


Fig. 8.3. The absorption coefficient of the negative hydrogen ion shows a maximum near 8500 Å. Two calculations are compared.

H^- f-f cross sections

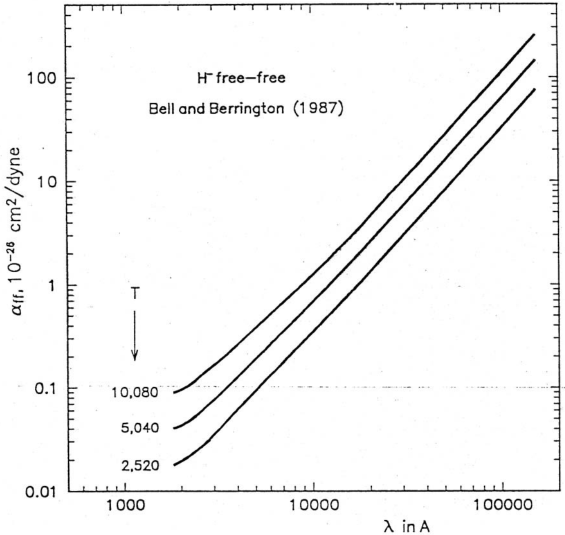


Fig. 8.4. The free–free absorption coefficient of the negative hydrogen ion increases with wavelength.

He^- f-f cross sections

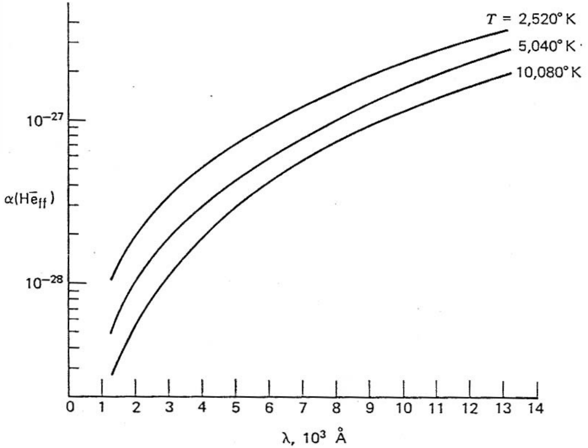
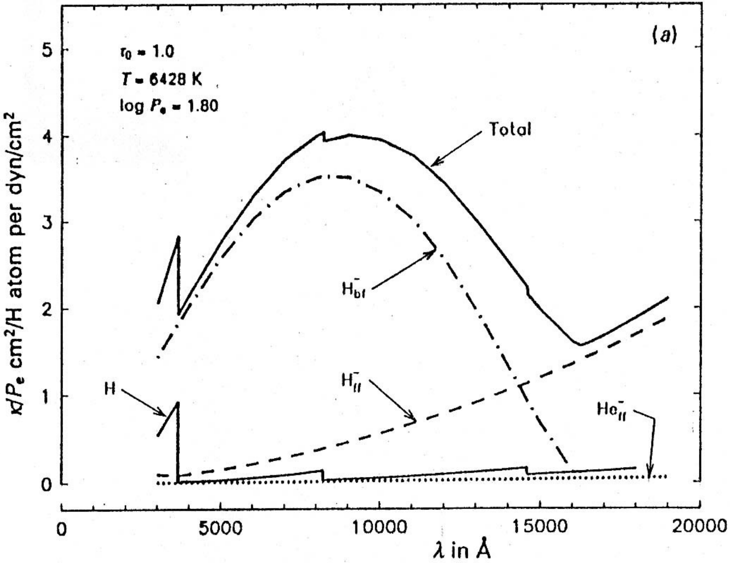
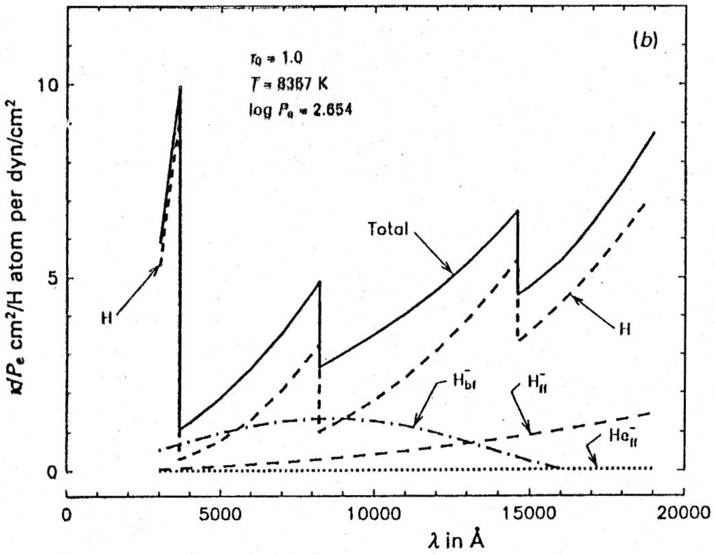


Fig. 8.5. The free–free absorption coefficient of the negative helium ion qualitatively mimics the absorption of the H^- ion. The units of $\alpha(He_{ff}^-)$ are square centimeters per helium atom per unit electron pressure.

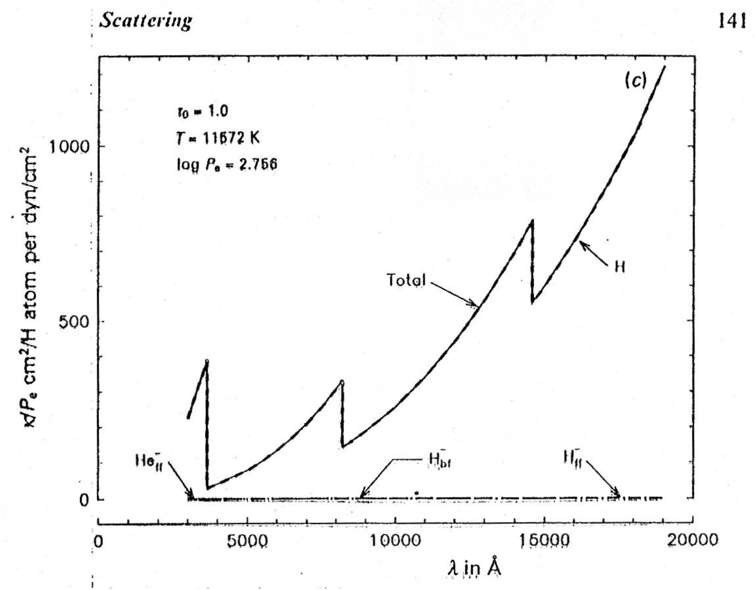
hydrogen opacities



hydrogen opacities



hydrogen opacities



metal b-f opacities

- metals = everything with $Z > 2$
- extremely important b-f and f-f opacity sources
- UV wavelengths!
- most important elements: C, Si, Al, Mg, Fe
- few cases: cross sections from lab data
- most cases: theoretical model calculations
- 1987: 'opacity project' (Seaton), OPAL (LLNL)
- $\rightarrow$ generate state-of-the-art databases of cross-sections
- assumes LS coupling to use quantum defect methods

electron scattering

- Thompson scattering: electron scattering in the non-relativistic limit
- $\rightarrow$ independent of λ:

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \approx 0.66 \times 10^{-24}\text{cm}^2$$

- phase function:

$$\propto 1 + \cos^2\theta$$

- $\rightarrow$ can be approximately treated as isotropic

electron scattering

- Thompson scattering is important if there are lots of free electrons
- $\rightarrow$ high T, small P_{gas}
- $\rightarrow$ relatively more important in giants compared to dwarfs

Rayleigh scattering

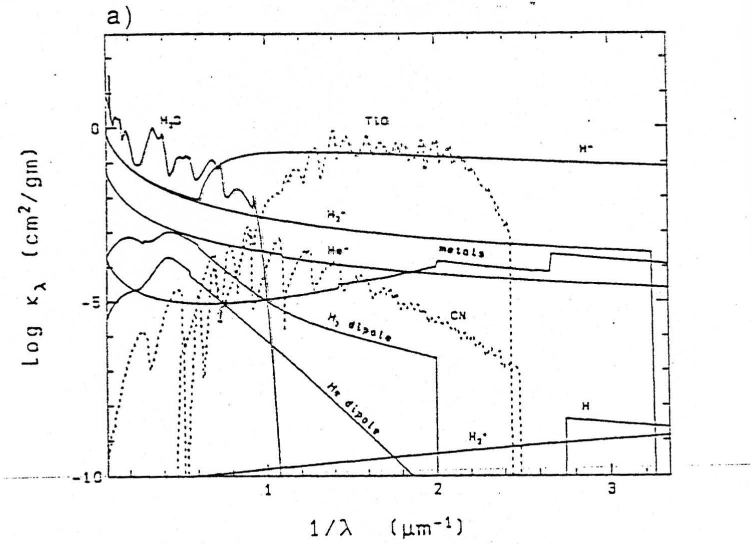
- important in cool stars
- important in the UV
- important for H I, H_2 and He I
- wavelength dependence of the form

$$\sigma_R(\lambda) \propto \frac{\lambda_0}{\lambda^4}\left[1 + \left(\frac{\lambda_2}{\lambda}\right)^2 + \left(\frac{\lambda_4}{\lambda}\right)^4\right]$$

molecular opacities

- cool stars
- mostly contribute to IR opacity
- these are spectral lines
- but *many* of them
- $\rightarrow$ overlap to form broad bands
- most important ones: TiO, H_2O, CO

molecular opacities



molecular opacities

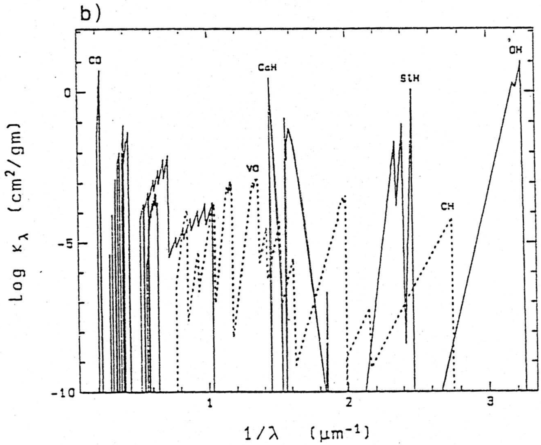
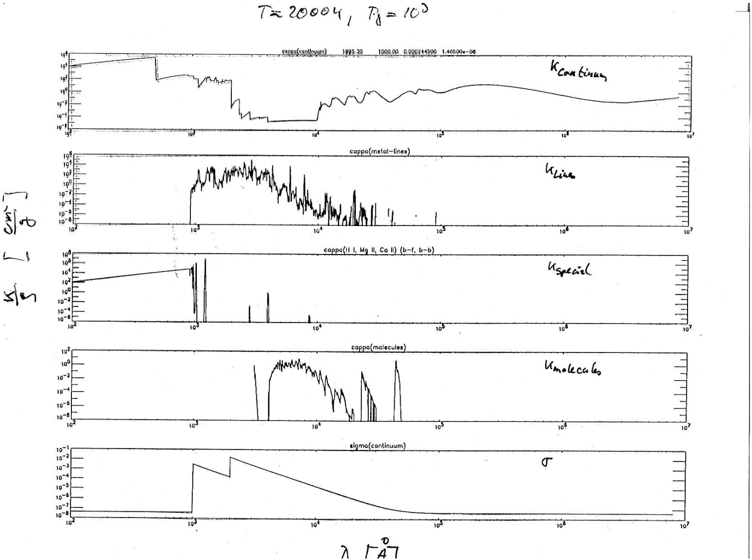
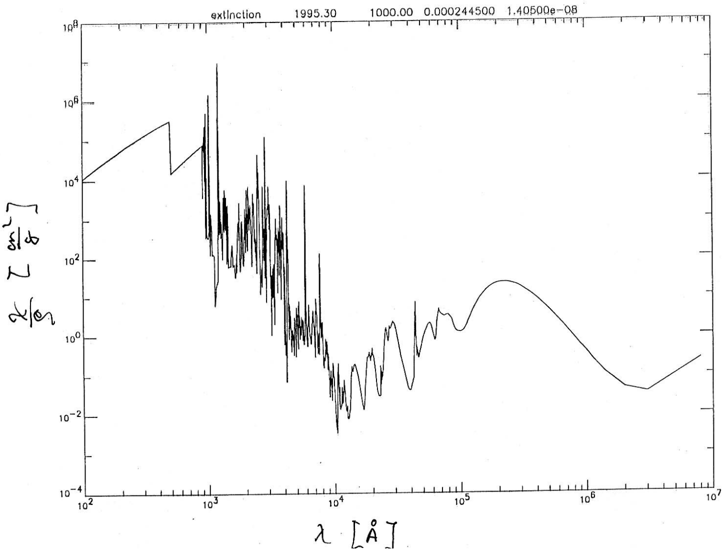


Figure 13: Wavelength distribution of the absorption coefficients in units of cm^2/g obtained at $\tau_{std} = 1$ in a model with $T_{eff} = 3000$ K, $\log g = 5.0$ and solar composition for a) the continuum opacity sources (—) and the molecular bands of CN and TiO (- - -) and for b) the molecular bands of the species taken into account in the JOLA (here VO and CH are represented with broken lines for more clarity).

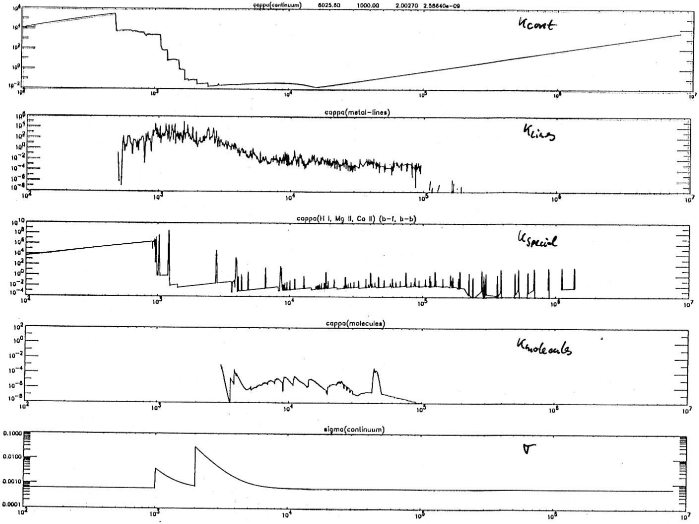
total opacities



total opacities



total opacities



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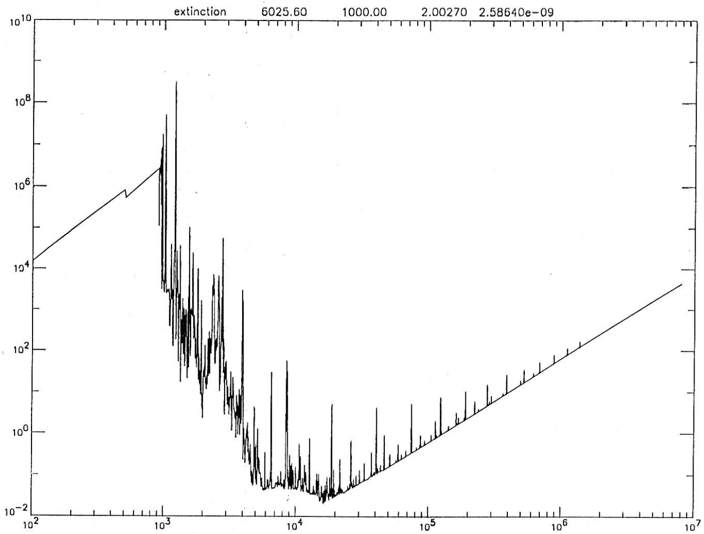

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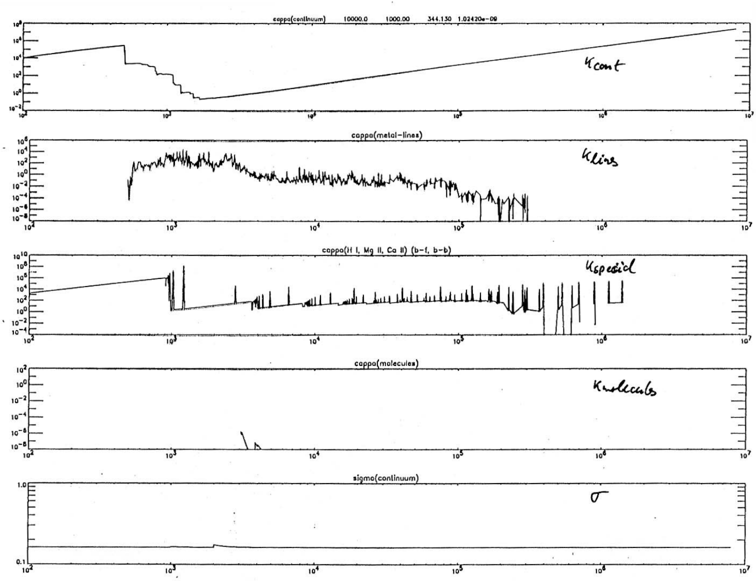

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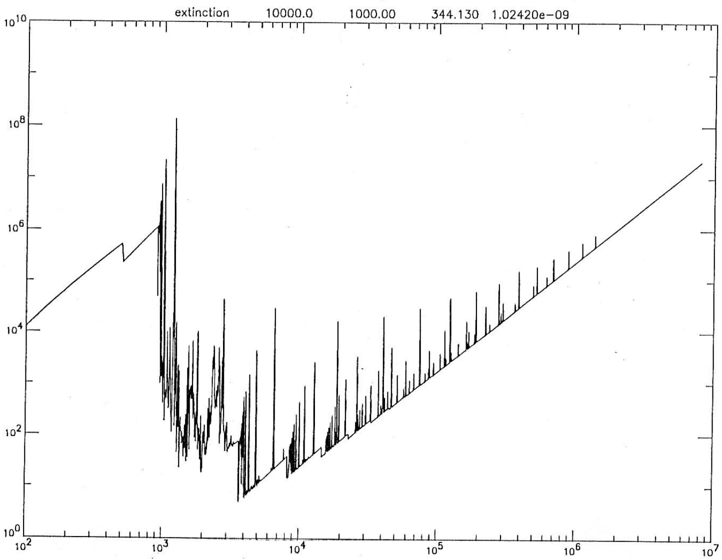

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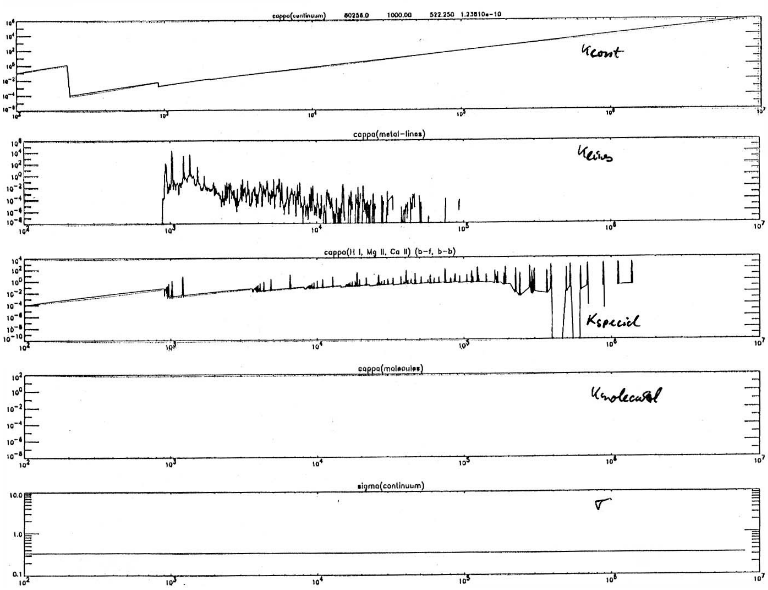

total opacities