Stellar/Planetary Atmospheres

Part 04: the scattering problem

Peter Hauschildt
yeti@hs.uni-hamburg.de

Hamburger Sternwarte
Gojenbergsweg 112
21029 Hamburg

14. März 2018

Topics

- The scattering problem
- Λ iteration problem

The scattering problem

- scattering is the *fundamental* physical problem in RT
- scattering couples different spatial locations of the atmosphere
- $\rightarrow$ global transport of photons over large distances
- causes large differences between B_ν and J_ν even for $\tau_\nu \gg 1$

The scattering problem

- consider

$$S = (1-\epsilon)J + \epsilon B$$

- the FS of the pp RTE

$$\mu \frac{dI}{d\tau} = I - S$$

can be written with the Λ operator

$$J(\tau) = \Lambda_\tau[S] = \Lambda_\tau[\epsilon B] + \Lambda_\tau[(1-\epsilon)J]$$

- $\epsilon = 1 \rightarrow$ no problem, direct solution
- $\epsilon = 0 \rightarrow$ grey atmosphere solution
- consider $0 < \epsilon < 1$

Λ iteration

- idea: fix-point iteration
- start with

  $$J = B$$

  as initial guess
- iterate:

  $$J^{(n)} = \Lambda_\tau[S^{(n-1)}] = \Lambda_\tau[\epsilon B] + \Lambda_\tau[(1-\epsilon)J^{(n-1)}]$$

- repeat until convergence, e.g.,

  $$|J^{(n)} - J^{(n-1)}| \ll 1$$

Λ iteration

- works mathematically (Λ is contracting)
- problem: fails in praxis if $\epsilon \ll 1$!
- why?
- consider linear (in τ) Planck function:

$$B(\tau) = a + b\tau$$

Λ iteration

- H is determined by

$$\frac{dH}{d\tau} = J - S = \epsilon(J - B)$$

- K is determined by

$$\frac{dK}{d\tau} = H$$

Λ iteration

- use Eddington approximation

$$K = \frac{1}{3} J$$

- substitute $\frac{dK}{d\tau} = H$ into $\frac{dH}{d\tau} \rightarrow$

$$\frac{1}{3}\frac{d^2 J}{d\tau^2} = \epsilon(J - B) = \frac{1}{3}\frac{d^2(J - B)}{d\tau^2}$$

Λ iteration

- for $\tau \to \infty$ we must have $J \to B$
- $\to$ solution

$$J(\tau) = a + b\tau + (b - \sqrt{3}a)\exp\left(-\sqrt{3\epsilon}\tau\right)\frac{1}{\sqrt{3} + \sqrt{3\epsilon}}$$

- this shows the essential physics:

Λ iteration

- J may be very different from B for small τ:
  - set $b = 0$ so that $a = B = \mathrm{const.}$

$$J(0) = \frac{\sqrt{\epsilon}}{1 + \sqrt{\epsilon}} B$$

  - if $\epsilon \ll 1$ then $J(0) \ll B$

Λ iteration

- $J \rightarrow B$ only for *very* large τ:
  - only if $\tau >> 1/\sqrt{\epsilon} \gg 1$
- $1/\sqrt{\epsilon}$ is called the *thermalization depth*

Λ iteration

- caused by ϵ being the photon destruction probability
- to ensure thermalization, a photon needs to be scattered $\approx 1/\epsilon$ times
- $\rightarrow$ it will random walk a large optical distance without destruction
- $\rightarrow$ coupling between different regions
- $\tau < 1/\sqrt{\epsilon} \rightarrow$ photon has a chance to escape through the surface
- $J < B$!

Λ iteration

- Λ is given by

$$\Lambda_\tau[S(t)] = \frac{1}{2}\int_0^\infty E_1(|t-\tau|)S(t)\,dt$$

- for large $\Delta\tau$

$$E_1(\Delta\tau) \approx \frac{\exp(-\Delta\tau)}{\Delta\tau}$$

- therefore, Λ propagates information over a distance of $\Delta\tau \approx 1$

Λ iteration

- start iteration with $J = B \rightarrow$
- need $\approx 1/\sqrt{\epsilon}$ iterations to reach outer boundary
- $\epsilon = 10^{-8} \rightarrow 10^4$ iterations
- in praxis, corrections $J^{(n)} - J^{(n-1)}$ tend to stabilize at small values
- $\rightarrow$ apparent convergence although still far (dex) from solution