Stellar/Planetary Atmospheres

Part 0

Peter Hauschildt
`yeti@hs.uni-hamburg.de`

Hamburger Sternwarte
Gojenbergsweg 112
21029 Hamburg

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Allgemeines/Organisation

- Wie kann man mich erreichen:
    - Email: `yeti@hs.uni-hamburg.de`
    - Tel: 040 428 38 - 8512
    - nach der Vorlesung
    - an der Sternwarte (bitte vorher Termin vereinbaren!)
- Bei Unklarheiten bitte *sofort* fragen!

Allgemeines/Organisation

- Vorkenntnisse:
    - Mathe/Physik: BSc. Physik
    - (good) Astro: Einführung in die Astronomie I+II
    - (helpful) Computer: Unix, Programmiersprache (Fortran 2008, C, C++, Perl, Python, Julia)
- Format: Wöchentliche Vorlesung (3SWS) + 1SWS Übungen

Leistungsnachweis

- Abschlussklausur (take home exam)/mündliche Prüfung
- mind. 50% der möglichen Punkte aus der Klausur

Allgemeines/Organisation

- Script: Wird auf dem Web zum Herunterladen vor der Vorlesung bereitgestellt
- `http://hobbes.hs.uni-hamburg.de/~yeti/`
    - User Name: `HHstudent`
    - Password: `phoenix`
- the script will be in English ...

Allgemeines/Organisation

- text books
  - D.F. Gray: 'The observation and analysis of stellar photospheres', CAS 20, Camb. Univ. Press, 1990
  - A. Unsöld: 'Physik der Sternatmosphären', 2nd Ed., Springer, 1955
  - D. Mihalas: 'Stellar Atmospheres', 1st Ed., Freeman 1970, 2nd Ed., Freeman 1978
  - Rutten's script: hunt for it on the Web!

Topics

- What do we want to do? Model Atmospheres!
- What do we need? Basic equations!
- description of radiation
- radiative transfer
- radiative equilibrium

Topics

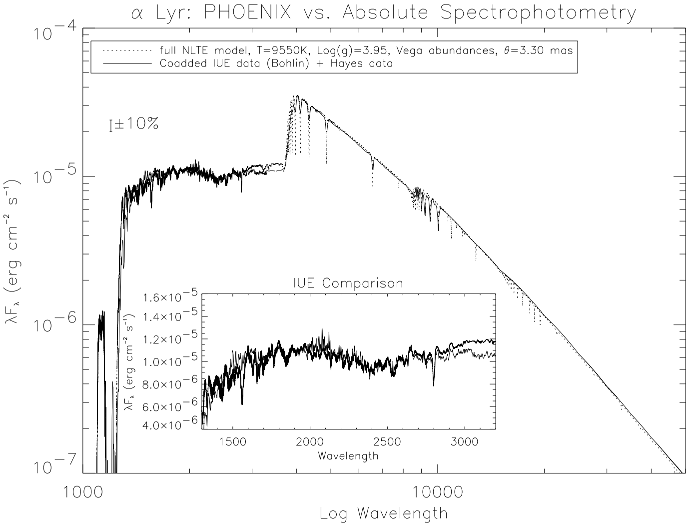
- LTE Equation of State (EOS)
- continuum absorption & emission
- line absorption & emission
- convective energy transport
- model atmospheres
- analysis of stellar atmospheres
- irradiated atmospheres (planets)
- Novae/Supernovae/Stellar winds

what can be done?

- construct detailed computer simulations of stellar & planetary atmospheres
- compare synthetic to observed spectra
- examples:

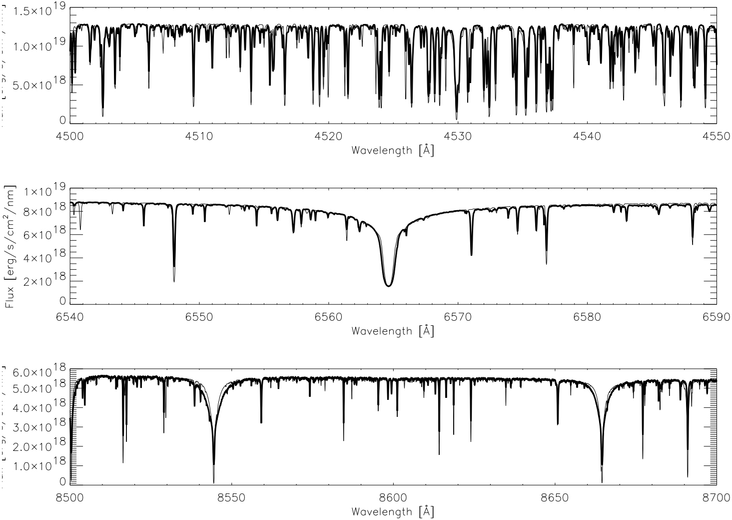
Example: A stars

- models work well for A0V's . . .



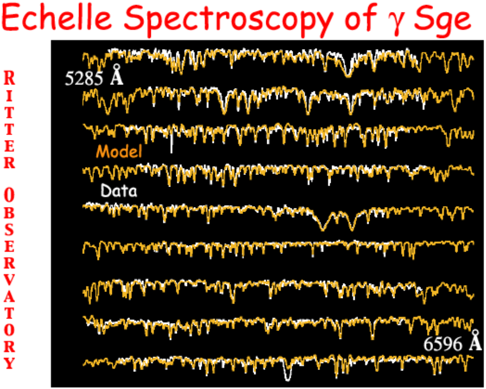
Example: solar type stars

- models work well for G2V's ...



Example: γ Sge

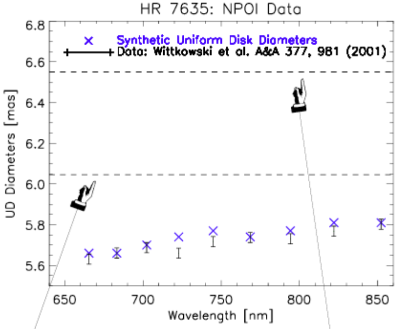
- `PHOENIX` model fit (Aufdenberg et al):



Example: γ Sge

- `PHOENIX` model fit (Aufdenberg et al):

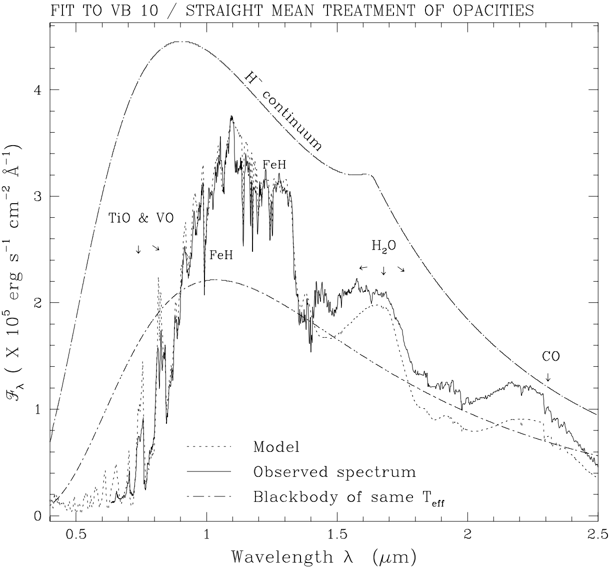
NPOI Wavelength Dependent Uniform Disk Sizes
for γ Sagittae
Measurements versus Model



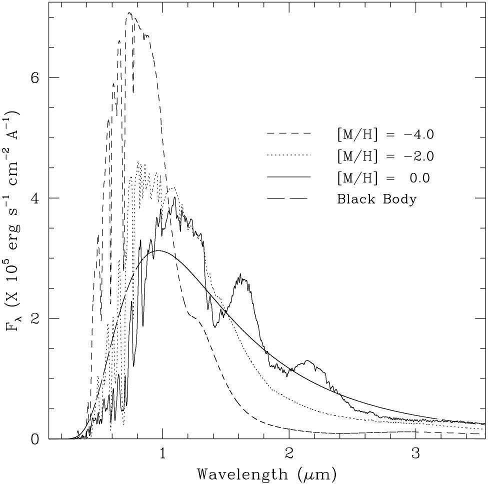
Spherical Model
Rosseland Diameter
(Optical Depth ≅ 1)

Spherical Model
Physical Diameter
(Zero Intensity)

Example: Cool Atmospheres

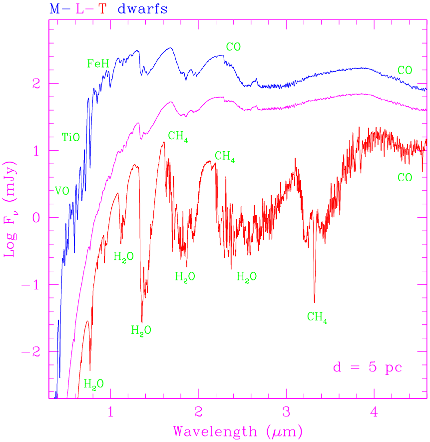


Example: Cool Atmospheres

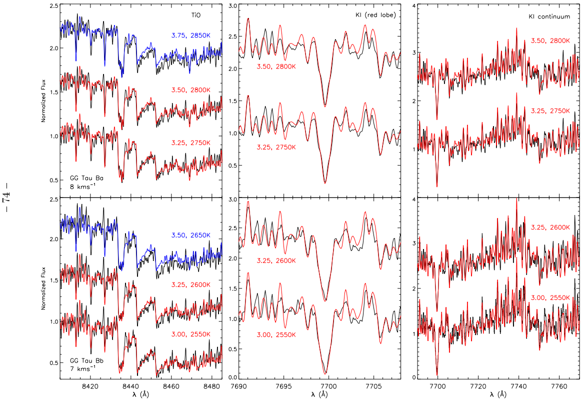


Stellar → Planetary Atmospheres

- Trends (Allard et al, 2001)
  - $T_{\mathrm{eff}} = 2500, 1800, 1000\,\mathrm{K}$
  - age 5Gy (Chabrier et al, 2000)



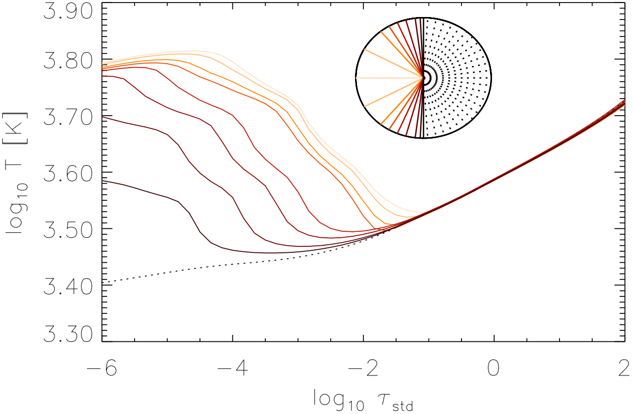
Example: GG Tau



Mohanty et al, ApJ

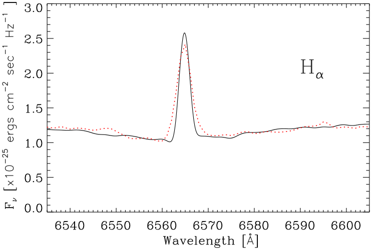
Example: Pre-CVs

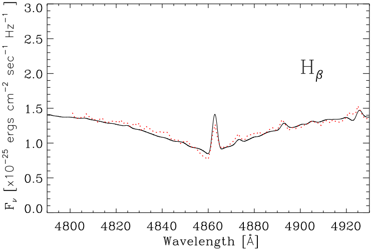
- Pre-CV: non-interacting WD+dM
- model dM irradiated by WD
  (Barman et al)



Example: GD 245

- observable emission lines from the dM





What is the atmosphere?

- outer region of the star
- transition from stellar interior to ISM
- connects the star to the 'outside world'
- all energy generated in the star has to pass through the atmosphere
- atmosphere itself usually does not produce additional energy!

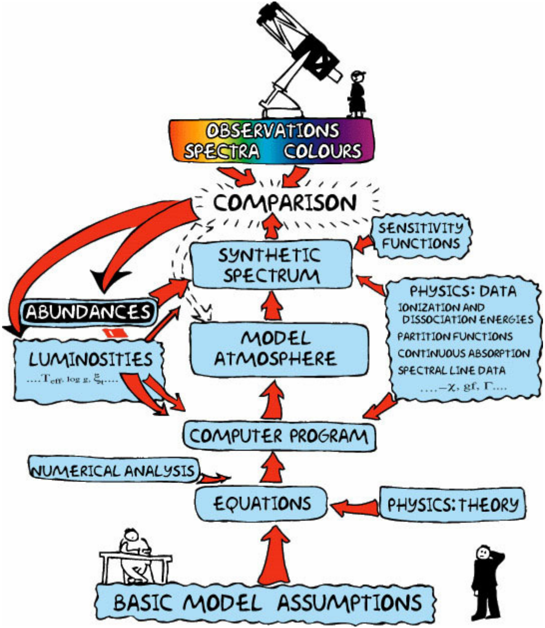
The photosphere

- region were most of the radiation escapes from the star
- 'only' part of the star that can be observed!
- see also:
  - chromosphere
  - corona

The photosphere

- some numbers:
  - Sun: $\Delta h \approx 1000\,\mathrm{km}$
  - white dwarf: $\Delta h \leq 100\,\mathrm{m}$
  - red giant: $\Delta h/R \approx 1.5$

What needs to be done?



Model Atmospheres

1. basic assumptions
2. equations
3. computation

Simplest Basic Assumptions

- 1D geometry: plane parallel or spherical
- hydrostatic equilibrium (but: stellar winds!)
- all surface structure ignored (starspots, granulation, activity)
- no magnetic fields (they are evil!)
- single object (but: planets!)

Hydrostatic Equilibrium

- *total* pressure gradient

$$\frac{dP}{dr} = -g(r)\rho$$

  - $g(r)$: gravity ($= \mathrm{const.}$ in plane parallel atm.)
  - ρ: density

Hydrostatic Equilibrium

- define 'standard' optical depth τ_{std}

$$d\tau_{\mathrm{std}} = -\chi_{\mathrm{std}} dr = -(\kappa_{\mathrm{std}} + \sigma_{\mathrm{std}}) dr$$

  - χ_{std}: 'standard' extinction coefficient
  - κ_{std}: 'standard' absorption coefficient
  - σ_{std}: 'standard' scattering coefficient
- 'standard' denotes suitable average etc.
- χ_{std} is a complicated function ...

Hydrostatic Equilibrium

- with this we have

$$\frac{dP}{d\tau_{\rm std}} = \frac{g(r)\rho}{\chi_{\rm std}}$$

- significant radiation pressure $P_{\rm rad} \rightarrow$

$$\frac{dP_{\rm gas}}{d\tau_{\rm std}} = \frac{g(r)\rho}{\chi_{\rm std}} - \frac{dP_{\rm rad}}{d\tau_{\rm std}}$$

  gives an expression for the gas pressure $P_{\rm gas}$

Insert: $dP_{\rm rad}/d\tau_{\rm std}$

- $dP_{\rm rad}/d\tau_{\rm std}$ is a function of the *opacity averaged radiation flux*

$$\frac{dP_{\rm rad}}{d\tau_{\rm std}} = \frac{\pi}{c}\frac{\int_0^\infty \chi_\lambda F_\lambda \, d\lambda}{\chi_{\rm std}}$$

  - F_λ: radiation flux

Hydrostatic Equilibrium

- solution $P(\tau_{\rm std})$ depends on
- $\chi_{\rm std}$ which is a function of
  - temperature
  - gas pressure (or density)
  - radiation field
- all of these are, initially, unknown
- $\rightarrow$ solution only by iteration etc.
- hydrostatic essentially determines $P_{\rm gas}(\tau_{\rm std})$

Energy Conservation

- plane parallel geometry
- all energy transported by radiation:

$$F_{\mathrm{rad}} = \int_0^\infty F_\lambda \, d\lambda \equiv \sigma T_{\mathrm{eff}}^4 = \mathrm{const.}$$

- equivalent (but numerically different!) condition:

$$\frac{dF_{\mathrm{rad}}}{d\tau_{\mathrm{std}}} = 0$$

Energy Conservation

- → each volume element has emission = absorption

$$\int_0^\infty \kappa(\lambda)\,(J_\lambda - S_\lambda)\;d\lambda = 0$$

  - J_λ: mean intensity (direction averaged)
  - S_λ: source function (simplest case: Planck-function)

Energy Conservation

- if *convection* is important we have

$$F_{\mathrm{total}} = F_{\mathrm{rad}} + F_{\mathrm{conv}} = \sigma T_{\mathrm{eff}}^4$$

  - F_{conv}: convective flux
- problem: no real theory to compute F_{conv}!

Energy Conservation

- energy conservation 'essentially' determines the $T(\tau_{\rm std})$ structure
- problem: need to know radiation flux!
- $\rightarrow$ integral over radiation field
- $\rightarrow$ need to know the whole radiation field!
- $\rightarrow$ need to solve a number of

Auxiliary Equations

- convective energy transport prescription
  - mixing length theory
- radiative transfer equation $\forall\lambda$
  - $\rightarrow$ continuum absorption and scattering
  - $\rightarrow$ spectral line absorption and scattering
    - $\rightarrow$ line profiles
- depend all on T, $P_{\rm gas}$, chemical abundances, radiation field

Auxiliary Equations

- absorption & scattering coefficients

$$\sum \sigma_i^j n_i^j$$

  - j: ionization stage
  - i: energy level within each ionization stage
  - σ_i^j: cross section [cm^2]
  - n_i^j: population density [1/cm^3]
- $\sum$ over all elements, processes, ionization stages, level
- σ_i^j from QM, measurements
- $\rightarrow$ tables of data, fit formulae etc.

Auxiliary Equations

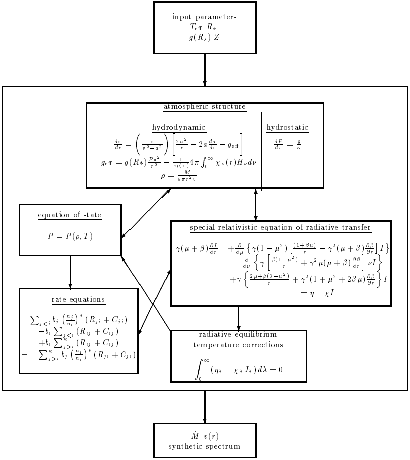
- n_i^j depend on
  - temperature
  - gas pressure
  - abundances
  - *radiation field* → 'NLTE' (*evil!!*)
- need to solve equation of state!
- gives relation $(T, P_{\rm gas}, \rho)$
- gives all n_i^j

Computation of Model Atmospheres

- analytic solutions only for idealized cases
- numerical solutions require iterations
- approximate solution (e.g., scaled semi-empirical)
  → useful starting guesses

Dependency Chart

PHOENIX REALWIND



input parameters

$T_{\rm eff}$ R_*

$g(R_*)$ Z

atmospheric structure

hydrodynamic

$$\frac{dv}{dr} = \left(\frac{v}{v^2 - a^2}\right)\left[\frac{2a^2}{r} - 2a\frac{da}{dr} - g_{\rm eff}\right]$$

$$g_{\rm eff} = g(R*)\frac{R*^2}{r^2} - \frac{1}{c\rho(r)}4\pi\int_0^\infty \chi_\nu(r)H_\nu d\nu$$

$$\rho = \frac{M}{4\pi r^2 v}$$

hydrostatic

$$\frac{dP}{d\tau} = \frac{g}{\kappa}$$

equation of state

$$P = P(\rho, T)$$

special relativistic equation of radiative transfer

$$\begin{aligned}\gamma(\mu+\beta)\frac{\partial I}{\partial r} \quad & +\frac{\partial}{\partial \mu}\left\{\gamma(1-\mu^2)\left[\frac{(1+\beta\mu)}{r} - \gamma^2(\mu+\beta)\frac{\partial \beta}{\partial r}\right] I\right\} \\ & -\frac{\partial}{\partial \nu}\left\{\gamma\left[\frac{\beta(1-\mu^2)}{r} + \gamma^2\mu(\mu+\beta)\frac{\partial \beta}{\partial r}\right]\nu I\right\} \\ & +\gamma\left\{\frac{2\mu+\beta(3-\mu^2)}{r} + \gamma^2(1+\mu^2+2\beta\mu)\frac{\partial \beta}{\partial r}\right\} I \\ & = \eta - \chi I\end{aligned}$$

rate equations

$$\begin{aligned}&\sum_{j<i} b_j \left(\frac{n_j}{n_i}\right)^* (R_{ji} + C_{ji}) \\ &-b_i \sum_{j<i} (R_{ij} + C_{ij}) \\ &+b_i \sum_{j>i}^{\kappa} (R_{ij} + C_{ij}) \\ &= -\sum_{j>i}^{\kappa} b_j \left(\frac{n_j}{n_i}\right)^* (R_{ji} + C_{ji})\end{aligned}$$

radiative equilibrium

temperature corrections

$$\int_0^\infty (\eta_\lambda - \chi_\lambda J_\lambda)\, d\lambda = 0$$

$\dot{M}, v(r)$

synthetic spectrum

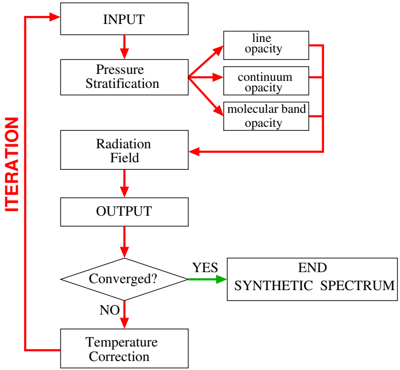
Model Atmospheres

- there are *multiple* ways to compute model atmospheres!
- I'll describe the one I use . . .
- Consult the literature for alternatives!

Model Atmospheres

- minimum independent variables/parameters:
  - effective temperature T_{eff}
  - gravity $g(r) = GM/r^2$
  - mass M or radius R or luminosity $L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$
  - abundances of all elements
- additional parameters may exist (B-fields, irradiation etc)

Model Atmosphere Flow Chart



Model Atmospheres

- necessary prelims:
  - discretize the radial coordinate
  - chose an independent variable representing r
    - r, but that's not convenient
    - column density (hydrostatic trivial!)
    - τ_{std}
      (very good for radiation transport)
  - I'll use τ_{std}!

Model Atmospheres

- step 0:
  - select a grid of $\tau_{\rm std}$ points
  - guess a temperature structure $T(\tau_{\rm std})$
  - integrate hydrostatic equation

$$\frac{dP_{\rm gas}}{d\tau_{\rm std}} = \frac{g(r)\rho}{\chi_{\rm std}}$$

  (ignoring $P_{\rm rad}$ for simplicity)
- plane-parallel geometry ($r \to z$):
  - let z increase inwards
  - $z = 0$ is outermost point
  - set $g = \mathrm{const.}$

Model Atmospheres

- need initial value for $P_{\mathrm{gas}}(\tau_{\mathrm{std}} = 0)$!
- need $\rho(T, P_{\mathrm{gas}}) \rightarrow$ equation of state, e.g.,

$$\rho = \frac{\mu}{\mathcal{R}} \frac{P_{\mathrm{gas}}}{T}$$

- $\mathcal{R} = k/m_{\mathrm{H}}$
- μ: mean molecular weight

Model Atmospheres

- step 1:
  - compute total radiative flux

    $$F_{\mathrm{rad}}(\tau_{\mathrm{std}}) = \int_0^{\infty} F_\lambda(\lambda, \tau_{\mathrm{std}})\, d\lambda$$

    for *each* layer!
  - → need to know $F_\lambda(\lambda)$ …
  - → need to solve radiative transfer problem $\forall\lambda$
  - → must know all $\sigma_i^j(T, P_{\mathrm{gas}}, \lambda)$
  - *and* must know all $n_i^j(\tau_{\mathrm{std}})$

Model Atmospheres

- step 2:
  - in general we will find that

$$F_{\mathrm{rad}}(\tau_{\mathrm{std}}) \neq F_{\mathrm{total}}$$

  - $\rightarrow$ need to *correct* $T(\tau_{\mathrm{std}})$
- repeat until converged . . .